Exercise 15

Find the derivative of the function.

$$f(t) = e^{at} \sin bt$$

Solution

Take the derivative using the product rule and the chain rule.

$$
\begin{aligned}
f'(t) = \frac{df}{dt} &= \frac{d}{dt}(e^{at} \sin bt) \\
&= \left[\frac{d}{dt}(e^{at})\right] \sin bt + e^{at} \left[\frac{d}{dt}(\sin bt)\right] \\
&= \left[e^{at} \cdot \frac{d}{dt}(at)\right] \sin bt + e^{at} \left[(\cos bt) \cdot \frac{d}{dt}(bt)\right] \\
&= \left[e^{at} \cdot (a)\right] \sin bt + e^{at} \left[(\cos bt) \cdot (b)\right] \\
&= ae^{at} \sin bt + be^{at} \cos bt \\
&= (a \sin bt + b \cos bt)e^{at}
\end{aligned}
$$